Review Problems:

1.28

1.63

3.20

3.46 a) & b)

3.113

3.125

1) Consider a plane wall that is insulated on the inner wall, while the outer wall is held at a constant temperature of 300K. The wall is 20 cm thick and made of Stainless Steel. The volumetric heat generation in the wall is given by the following expression $\dot{q} = Q \cdot x$, where Q = 1000 W/m^4.

a) Derive and expression for the steady state temperature profile within the wall.
b) Calculate the heat flux at each wall

2) Compare the effectiveness and the efficiency of a rectangular versus a cylindrical fin. Both fins are the same length of 40 cm and are made of pure copper. The cylindrical fin has a diameter of 10 cm, while the rectangular fin has a thickness of 2 cm and a width of 13.72 cm. The heat transfer coefficient, h = 10 W/m2K, the base temperature of the fin is 50 C while the air is at 20 C.

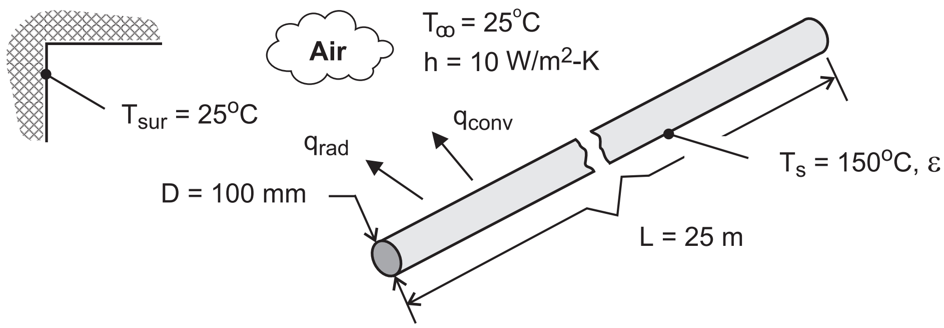
a) Calculate the effectiveness and efficiency of both fins assuming an adiabatic tip.
b) Calculate the heat loss through the cylindrical fin
c) Calculate the temperature at the tip of the cylindrical fin.

PROBLEM 1.28

KNOWN: Length, diameter, surface temperature and emissivity of steam line. Temperature and convection coefficient associated with ambient air. Efficiency and fuel cost for gas fired furnace.

FIND: (a) Rate of heat loss, (b) Annual cost of heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steam line operates continuously throughout year, (2) Net radiation transfer is between small surface (steam line) and large enclosure (plant walls).

ANALYSIS: (a) From Eqs. (1.3a) and (1.7), the heat loss is

$$q = q_{conv} + q_{rad} = A\left[h\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

where $A = \pi DL = \pi\left(0.1m \times 25m\right) = 7.85m^2.$

Hence,

$$q = 7.85m^2\left[10\ W/m^2 \cdot K\left(150 - 25\right)K + 0.8 \times 5.67 \times 10^{-8} W/m^2 \cdot K^4\left(423^4 - 298^4\right)K^4\right]$$

$$q = 7.85m^2\left(1,250 + 1,095\right)w/m^2 = \left(9813 + 8592\right)W = 18,405\ W$$ <

(b) The annual energy loss is

$$E = qt = 18,405\ W \times 3600\ s/h \times 24h/d \times 365\ d/y = 5.80 \times 10^{11}\ J$$

With a furnace energy consumption of $E_f = E/\eta_f = 6.45 \times 10^{11}\ J,$ the annual cost of the loss is

$$C = C_g E_f = 0.01\ \$/MJ \times 6.45 \times 10^5 MJ = \$6450$$ <

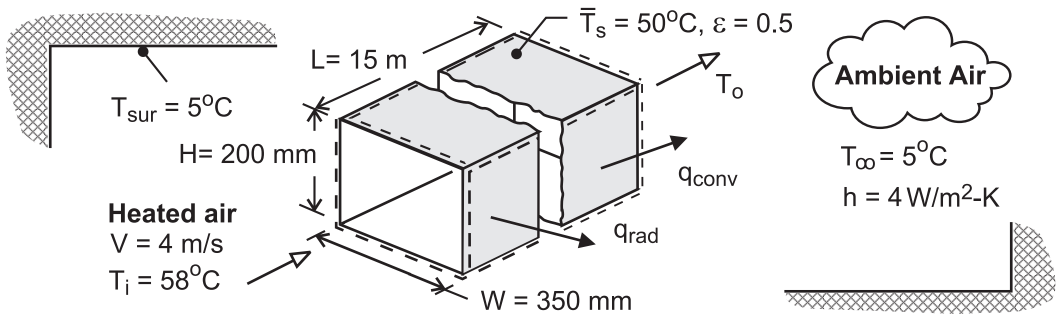
COMMENTS: The heat loss and related costs are unacceptable and should be reduced by insulating the steam line.

PROBLEM 1.63

KNOWN: Dimensions, average surface temperature and emissivity of heating duct. Duct air inlet temperature and velocity. Temperature of ambient air and surroundings. Convection coefficient.

FIND: (a) Heat loss from duct, (b) Air outlet temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Constant air properties, (3) Negligible potential and kinetic energy changes of air flow, (4) Radiation exchange between a small surface and a large enclosure.

ANALYSIS: (a) Heat transfer from the surface of the duct to the ambient air and the surroundings is given by Eq. (1.10)

$$q = hA_s\left(T_s - T_\infty\right) + \varepsilon A_s \sigma\left(T_s^4 - T_{sur}^4\right)$$

where A_s = L (2W + 2H) = 15 m (0.7 m + 0.5 m) = 16.5 m^2. Hence,

$$q = 4\ \text{W/m}^2\cdot\text{K}\times 16.5\ \text{m}^2\left(45^\circ\text{C}\right) + 0.5\times 16.5\ \text{m}^2\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(323^4 - 278^4\right)\text{K}^4$$

$$q = q_{conv} + q_{rad} = 2970\ \text{W} + 2298\ \text{W} = 5268\ \text{W}$$ <

(b) With i = u + pv, $\dot{W}$ = 0 and the third assumption, Eq. (1.11e) yields,

$$\dot{m}\left(i_i - i_o\right) = \dot{m}c_p\left(T_i - T_o\right) = q$$

where the sign on q has been reversed to reflect the fact that heat transfer is *from* the system.

With $\dot{m} = \rho V A_c = 1.10\ \text{kg/m}^3 \times 4\ \text{m/s}\left(0.35\text{m}\times 0.20\text{m}\right) = 0.308\ \text{kg/s},$ the outlet temperature is

$$T_o = T_i - \frac{q}{\dot{m}c_p} = 58^\circ\text{C} - \frac{5268\ \text{W}}{0.308\ \text{kg/s}\times 1008\ \text{J/kg}\cdot\text{K}} = 41^\circ\text{C}$$ <

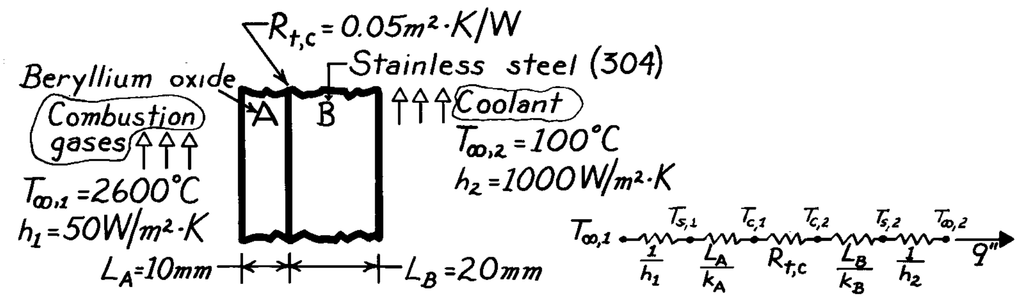
COMMENTS: The temperature drop of the air is large and unacceptable, unless the intent is to use the duct to heat the basement. If not, the duct should be insulated to insure maximum delivery of thermal energy to the intended space(s).

PROBLEM 3.20

KNOWN: Materials and dimensions of a composite wall separating a combustion gas from a liquid coolant.

FIND: (a) Heat loss per unit area, and (b) Temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Constant properties, (4) Negligible radiation effects.

PROPERTIES: *Table A-1*, St. St. (304) $\left(\bar{T} \approx 1000K\right)$: k = 25.4 W/m·K; *Table A-2*, Beryllium Oxide (T ≈ 1500K): k = 21.5 W/m·K.

ANALYSIS: (a) The desired heat flux may be expressed as

$$q''=\frac{T_{\infty,1}-T_{\infty,2}}{\frac{1}{h_1}+\frac{L_A}{k_A}+R_{t,c}+\frac{L_B}{k_B}+\frac{1}{h_2}}=\frac{(2600-100)^\circ C}{\left[\frac{1}{50}+\frac{0.01}{21.5}+0.05+\frac{0.02}{25.4}+\frac{1}{1000}\right]\frac{m^2.K}{W}}$$

$$q''=34{,}600\ W/m^2.$$ <

(b) The composite surface temperatures may be obtained by applying appropriate rate equations. From the fact that $q''=h_1\left(T_{\infty,1}-T_{s,1}\right)$, it follows that

$$T_{s,1}=T_{\infty,1}-\frac{q''}{h_1}=2600^\circ C-\frac{34,600\ W/m^2}{50\ W/m^2\cdot K}1908^\circ C.$$

With $q''=\left(k_A/L_A\right)\left(T_{s,1}-T_{c,1}\right)$, it also follows that

$$T_{c,1}=T_{s,1}-\frac{L_A q''}{k_A}=1908^\circ C-\frac{0.01m\times 34{,}600\ W/m^2}{21.5\ W/m\cdot K}=1892^\circ C.$$

Similarly, with $q''=\left(T_{c,1}-T_{c,2}\right)/R_{t,c}$

$$T_{c,2}=T_{c,1}-R_{t,c}q''=1892^\circ C-0.05\frac{m^2\cdot K}{W}\times 34,600\frac{W}{m^2}=162^\circ C$$

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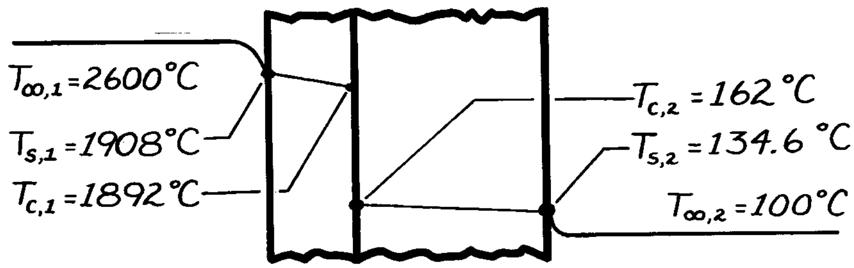
and with $q'' = (k_B / L_B)(T_{c,2} - T_{s,2})$,

$$T_{s,2} = T_{c,2} - \frac{L_B q''}{k_B} = 162^\circ C - \frac{0.02\,m \times 34{,}600\ W/m^2}{25.4\ W/m \cdot K} = 134.6^\circ C.$$

The temperature distribution is therefore of the following form:



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COMMENTS: (1) The calculations may be checked by recomputing q'' from

$$q'' = h_2 (T_{s,2} - T_{\infty,2}) = 1000\,W/m^2 \cdot K (134.6\text{-}100)^\circ C = 34{,}600\,W/m^2$$

(2) The initial *estimates* of the mean material temperatures are in error, particularly for the stainless steel. For improved accuracy the calculations should be repeated using k values corresponding to $T \approx 1900°C$ for the oxide and $T \approx 115°C$ for the steel.

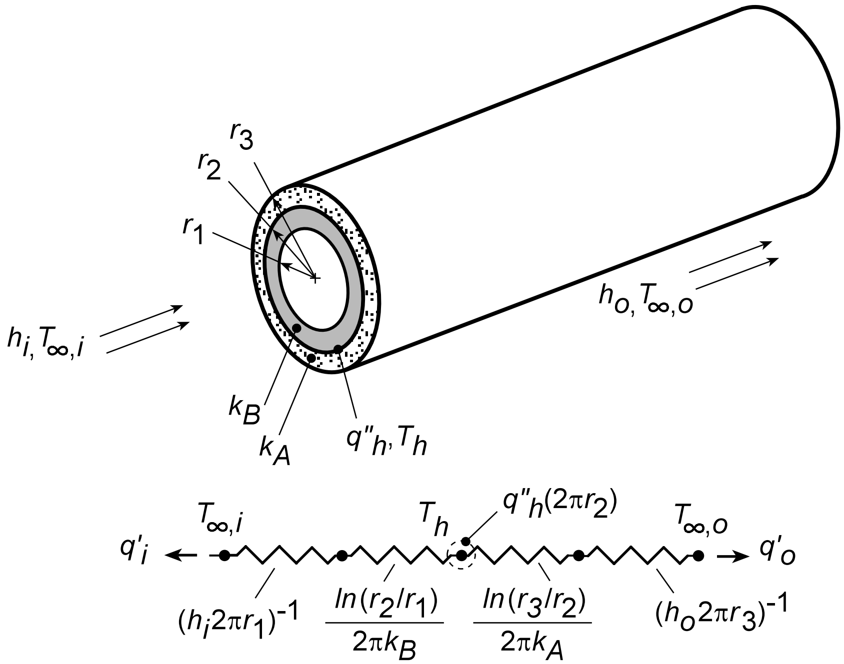
(3) The major contributions to the total resistance are made by the combustion gas boundary layer and the contact, where the temperature drops are largest.

PROBLEM 3.46

KNOWN: Conditions associated with a composite wall and a thin electric heater.

FIND: (a) Equivalent thermal circuit, (b) Expression for heater temperature, (c) Ratio of outer and inner heat flows and conditions for which ratio is minimized.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Constant properties, (3) Isothermal heater, (4) Negligible contact resistance(s).

ANALYSIS: (a) On the basis of a unit axial length, the circuit, thermal resistances, and heat rates are as shown in the schematic.

(b) Performing an energy balance for the heater, $\dot{E}_{in} = \dot{E}_{out}$, it follows that

$$q''_h(2\pi r_2) = q'_i + q'_o = \frac{T_h - T_{\infty,i}}{(h_i 2\pi r_1)^{-1} + \frac{\ln(r_2/r_1)}{2\pi k_B}} + \frac{T_h - T_{\infty,o}}{(h_o 2\pi r_3)^{-1} + \frac{\ln(r_3/r_2)}{2\pi k_A}} \qquad <$$

(c) From the circuit,

$$\frac{q'_o}{q'_i} = \frac{(T_h - T_{\infty,o})}{(T_h - T_{\infty,i})} \times \frac{(h_i 2\pi r_1)^{-1} + \frac{\ln(r_2/r_1)}{2\pi k_B}}{(h_o 2\pi r_3)^{-1} + \frac{\ln(r_3/r_2)}{2\pi k_A}} \qquad <$$

To reduce q'_o/q'_i, one could increase k_B, h_i, and r_3/r_2, while reducing k_A, h_o and r_2/r_1.

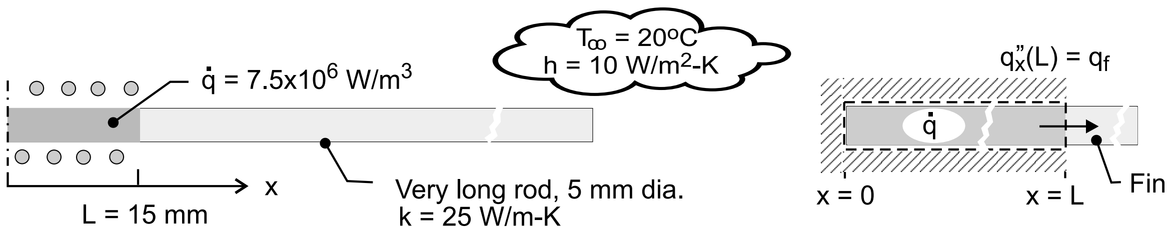
COMMENTS: Contact resistances between the heater and materials A and B could be important.

PROBLEM 3.113

KNOWN: Very long rod (D, k) subjected to induction heating experiences uniform volumetric generation $(\dot{q})$ over the center, 30-mm long portion. The unheated portions experience convection (T_∞, h).

FIND: Calculate the temperature of the rod at the mid-point of the heated portion within the coil, T_o, and at the edge of the heated portion, T_b.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with uniform $\dot{q}$ in portion of rod within the coil; no convection from lateral surface of rod, (3) Exposed portions of rod behave as infinitely long fins, and (4) Constant properties.

ANALYSIS: The portion of the rod within the coil, $0 \le x \le +L$, experiences one-dimensional conduction with uniform generation. From Eq. 3.43,

$$T_o = \frac{\dot{q}L^2}{2k} + T_b \tag{1}$$

The portion of the rod beyond the coil, $L \le x \le \infty$, behaves as an infinitely long fin for which the heat rate from Eq. 3.80 is

$$q_f = q_x(L) = (hPkA_c)^{1/2}(T_b - T_\infty) \tag{2}$$

where $P = \pi D$ and $A_c = \pi D^2/4$. From an overall energy balance on the imbedded portion of the rod as illustrated in the schematic above, find the heat rate as

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$$

$$-q_f + \dot{q}A_cL = 0$$

$$q_f = \dot{q}A_cL \tag{3}$$

Combining Eqs. (1-3),

$$T_b = T_\infty + \dot{q}A_c^{1/2}L(hPk)^{-1/2} \tag{4}$$

$$T_o = T_\infty + \frac{\dot{q}L^2}{2k} + \dot{q}A_c^{1/2}L(hPk)^{-1/2} \tag{5}$$

and substituting numerical values find

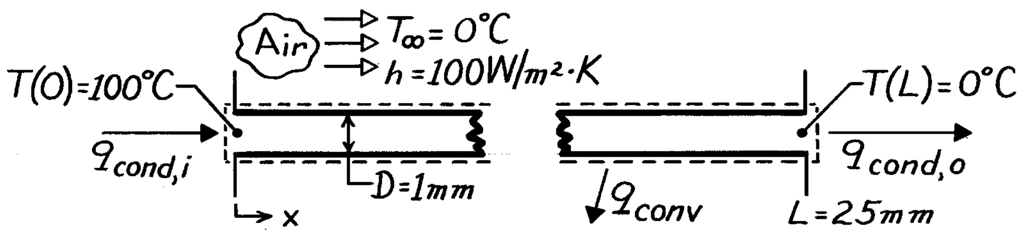
$$T_o = 305°C \qquad T_b = 272°C$$ <

PROBLEM 3.125

KNOWN: Dimensions and end temperatures of pin fins.

FIND: (a) Heat transfer by convection from a single fin and (b) Total heat transfer from a 1 m^2 surface with fins mounted on 4mm centers.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction along rod, (3) Constant properties, (4) No internal heat generation, (5) Negligible radiation.

PROPERTIES: *Table A-1,* Copper, pure (323K): k ≈ 400 W/m·K.

ANALYSIS: (a) By applying conservation of energy to the fin, it follows that

$$q_{conv} = q_{cond,i} - q_{cond,o}$$

where the conduction rates may be evaluated from knowledge of the temperature distribution. The general solution for the temperature distribution is

$$\theta(x) = C_1\, e^{mx} + C_2\, e^{-mx} \qquad \theta \equiv T - T_\infty.$$

The boundary conditions are $\theta(0) \equiv \theta_o$ = 100°C and $\theta(L) = 0$. Hence

$$\theta_o = C_1 + C_2$$

$$0 = C_1\, e^{mL} + C_2\, e^{-mL}$$

Therefore, $\quad C_2 = C_1\, e^{2mL}$

$$C_1 = \frac{\theta_o}{1-e^{2mL}}, \qquad C_2 = -\frac{\theta_o\, e^{2mL}}{1-e^{2mL}}$$

and the temperature distribution has the form

$$\theta = \frac{\theta_o}{1-e^{2mL}}\left[e^{mx} - e^{2mL\text{-}mx}\right].$$

The conduction heat rate can be evaluated by Fourier's law,

$$q_{cond} = -kA_c\frac{d\theta}{dx} = -\frac{kA_c\theta_o}{1-e^{2mL}}\, m\left[e^{mx} + e^{2mL\text{-}mx}\right]$$

or, with $m = (hP/kA_c)^{1/2}$,

$$q_{cond} = -\frac{\theta_o\,(hPkA_c)^{1/2}}{1-e^{2mL}}\left[e^{mx} + e^{2mL\text{-}mx}\right].$$

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Hence at x = 0,

$$q_{cond,i} = -\frac{\theta_o \left(hPkA_c\right)^{1/2}}{1-e^{2mL}}\left(1+e^{2mL}\right)$$

at x = L

$$q_{cond,o} = -\frac{\theta_o \left(hPkA_c\right)^{1/2}}{1-e^{2mL}}\left(2e^{mL}\right)$$

Evaluating the fin parameters:

$$m = \left[\frac{hP}{kA_c}\right]^{1/2} = \left[\frac{4h}{kD}\right]^{1/2} = \left[\frac{4\times 100\ W/m^2\cdot K}{400\ W/m\cdot K\times 0.001m}\right]^{1/2} = 31.62\ m^{-1}$$

$$\left(hPkA_c\right)^{1/2} = \left[\frac{\pi^2}{4}D^3hk\right]^{1/2} = \left[\frac{\pi^2}{4}\times\left(0.001m\right)^3\times 100\frac{W}{m^2\cdot K}\times 400\frac{W}{m\cdot K}\right]^{1/2} = 9.93\times 10^{-3}\ \frac{W}{K}$$

$$mL = 31.62\ m^{-1}\times 0.025m = 0.791, \qquad e^{mL} = 2.204, \qquad e^{2mL} = 4.865$$

The conduction heat rates are

$$q_{cond,i} = \frac{-100K\left(9.93\times 10^{-3}\ W/K\right)}{-3.865}\times 5.865 = 1.507\ W$$

$$q_{cond,o} = \frac{-100K\left(9.93\times 10^{-3}\ W/K\right)}{-3.865}\times 4.408 = 1.133\ W$$

and from the conservation relation,

$$q_{conv} = 1.507\ W - 1.133\ W = 0.374\ W. \qquad <$$

(b) The total heat transfer rate is the heat transfer from N = 250×250 = 62,500 rods and the heat transfer from the remaining (bare) surface ($A = 1m^2 - NA_c$). Hence,

$$q = N\ q_{cond,i} + hA\theta_o = 62,500\ \left(1.507\ W\right) + 100W/m^2\cdot K\left(0.951\ m^2\right)\ 100K$$

$$q = 9.42\times 10^4\ W + 0.95\times 10^4\ W = 1.037\times 10^5\ W.$$

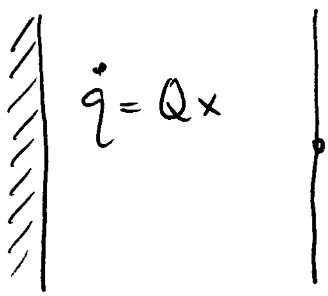
COMMENTS: (1) The fins, which cover only 5% of the surface area, provide for more than 90% of the heat transfer from the surface.

(2) The fin effectiveness, $\varepsilon \equiv q_{cond,i}/hA_c\theta_o$, is ε = 192, and the fin efficiency, $\eta \equiv \left(q_{conv}/h\pi\ DL\theta_o\right)$, is $\eta = 0.48$.

(3) The temperature distribution, $\theta(x)/\theta_o$, and the conduction term, $q_{cond,i}$, could have been obtained directly from Eqs. 3.77and 3.78, respectively.

(4) Heat transfer by convection from a single fin could also have been obtained from Eq. 3.73.

1) Consider a plane wall - Find the Temp Distribution



$q'' = 0$ insulated

$\therefore \left.\dfrac{dT}{dx}\right|_{x=0} = 0$

$T(L) = 300\,K$

Steady State

$$k\frac{d^2T}{dx^2} + \underbrace{\cancel{k\frac{d^2T}{dy^2}}}_{} ^{1D} + \cancel{k\frac{d^2T}{dz^2}}^{1D} + \dot{q} = \rho C_p \cancel{\frac{dT}{dt}}$$

$$\frac{d^2T}{dx^2} = -\frac{\dot{q}}{k} \qquad \dot{q} = Qx$$

$$\frac{d^2T}{dx^2} = -\frac{Qx}{k}$$

$$\frac{dT}{dx} = -\frac{Qx^2}{2k} + C_1$$

$$T(x) = -\frac{Qx^3}{6k} + C_1 x + C_2$$

$$\left.\frac{dT}{dx}\right|_{x=0} = C_1 = 0$$

$$T(L) = -\frac{QL^3}{6k} + C_2 = 300$$

$$C_2 = 300 + \frac{QL^3}{6k}$$

a) $$\boxed{T(x) = -\frac{Q}{6k}\left(x^3 - L^3\right) + 300}$$

b) Calculate heat flux at each wall

innerwall - insulated

$$\boxed{q'' = 0}$$

outer wall

$$q'' = -k \frac{dT}{dx}$$

$$q'' = -k\left(-\frac{Qx^2}{2k}\right)$$

$$\boxed{q'' = \frac{Qx^2}{2}}$$

2a) Compare the effectiveness and efficiency of rectangular versus cylindrical fin

$h = 20\ W/m^2K$ $\qquad$ $K = 401\ W/mK$

Cylindrical

$L = 40$ cm

$D = 10$ cm

$P = 0.314$ m

$A_c = 7.854 \times 10^{-3}\ m^2$

$$m = \sqrt{\frac{hP}{KA_c}}$$

$m = 1.412$

$$\varepsilon_f = \sqrt{\frac{KP}{hA_c}} \tanh mL$$

$\varepsilon_f = 14.5$

Rectangular

$L = 40$ cm

$t = 2$ cm

$w = 13.72$ cm

$P = 0.314$ m

$A_c = 0.002744\ m^2$

$$m = \sqrt{\frac{hP}{KA_c}}$$

$m = 2.388$

$$\varepsilon_f = \sqrt{\frac{KP}{hA_c}} \tanh mL$$

$\varepsilon_f = 35.5$

b) Calculate heat loss through cylindrical fin

$$q_f = \sqrt{hPKA_c}\ \tanh(mL)(T_b - T_\infty)$$

$q_f = 2.275\ \theta_b$ (W)

$q_f = 68.25 \text{ W}$

c) Calculate temp at the tip of the fin

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL}$$

$L = 0.4 \text{ m}$

$X = 0.4 \text{ m}$

$$\cosh(0) = 1$$

$$\frac{\theta}{\theta_b} = \frac{1}{\cosh mL}$$

$$T = \frac{(T_b - T_\infty)}{\cosh mL} + T_{inf}$$

$T = 45.8 \ ^\circ C$